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Code—14

MATHEMATICS

Time Allowed : 3 Hours *Maximum Marks : 150*

Note : Attempt any *Five* questions. All questions carry equal marks. Q. No. **1** is compulsory. Attempt *two* questions from Part I and *two* questions from Part II. The parts of the same question must be answered together and must not be interposed between answers to other questions.

1. Attempt any *four* of the following : (4×7½=30)

(a) Prove that any homomorphism from a finite dimensional vector space onto itself is an isomorphism.

(b) Find the area bounded by the curve :

$$y = x^3 - 6x^2 + 8x$$

and x-axis.

P.T.O.

(c) The extremities of a light pole rest on two smooth pegs A and B in the same horizontal line. A heavy load hangs from the point P of the pole. If AP = 3 PB and the pressure at B is 25N more than that at A, find the weight of the load.

(d) If p and q are chosen randomly from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ with replacement, find the probability that the roots of the equation :

$$x^2 + px + q$$

are real.

(e) Two bodies of mass 9 and 16 kg are placed at a distance of 10 m on a smooth table. If they attract each other with a constant force of one N, find after what time they will meet ?

(f) Let $f : [0, 1] \to \mathbf{R}$ be a continuous function. Prove that there exists $x \in [0, 1]$ such that $f(x) = x$.

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Part I

2. (a) Prove that if A is a $n \times n$-matrix with complex entries, then A can be expressed as $P + iQ$ where P and Q are Hermitian matrices. **(10)**

(b) Reduce the real quadratic form :

$$3x_1^2 - 3x_2^2 - 5x_3^2 - 2x_1 x_2 - 6x_2 x_3 - 6x_3 x_1$$

to the canonical form. Find its rank and index. **(20)**

3. (a) (i) Define the Gamma function. Prove that if n is a positive integer, then $\Gamma n = (n - 1)!$.

(ii) Find the maxima of :

$$u = x^2 + y^2 + z^2$$

where $2x^2 + 3y^2 + z^2 = 1$.

(5+10)

(b) (i) Find the equation of shortest distance between the lines :

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

and $\dfrac{x-2}{3} = \dfrac{y-4}{4} = \dfrac{z-5}{5}$.

(ii) Find the equation of cone with vertex at (1, 1, 1) and passing through the curve of intersection of :

$$x^2 + y^2 + z^2 = 1$$

and $$x + y + z = 1.$$

(7+8)

4. (a) Find the differential equation that represents all parabolas each of which has latus rectum $4a$ and whose axes are parallel to the x-axis. (8)

(b) Solve : (10)

$$\frac{dy}{dx} + y \tan x = y^3 \sec x.$$

(c) Solve : (12)

$$(x^7 y^2 + 3y)\, dx + (3x^8 y - x)\, dy = 0.$$

Part II

5. (a) Evaluate : (15)

$$\iint_S \left(x^3\, dy\, dz + x^2 y\, dz\, dx + x^2 z\, dy\, dx\right)$$

where S is the sphere $x^2 + y^2 + z^2 = 1$.

5\. (b) (i) Prove that for vectors A and B :

$$\nabla (A \cdot B) = (A \cdot \nabla) B + (B \cdot \nabla) A + A \times (\nabla \times B) + B \times (\nabla \times A)$$

(ii) Prove that for a vector F :

$$(\nabla \cdot \nabla) F = \nabla \cdot (\nabla F) = \nabla^2 F.$$

(10+5)

6\. (a) If f is continuous real valued function of real variable which satisfies $f(x + y) = f(x) \cdot f(y)$ for all x and y, then prove that either $f(x) = 0$ for all x or there is $a > 0$ such that $f(x) = a^x$.

(20)

(b) Show that the series :

$$\frac{x}{x+1} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \ldots\ldots\ldots$$

converges uniformly on $(K + \infty)$ where K is any positive integer. **(10)**

7. (a) Derive the Lagrange's interpolation formula to fit the data : **(10)**

x :	-1	1	2	3
$f(x)$:	1	1	4	9

(b) The median and mode of the following distribution are known to be 33·50 and 34 respectively. Find the values of f_3, f_4, f_5 given $\Sigma f = 230$. **(10)**

Class	f
0–10	4
10–20	16
20–30	f_3
30–40	f_4
40–50	f_5
50–60	6
60–70	4

(c) A ship is sailing westwards at 8 m/sec. While trying to fix a bolt at the top of the mast, the sailor drops the bolt. If the mast of the ship is 19·6 m high, where will the bolt hit the deck ? (Given g = 9·8 m/sec.) **(10)**