

Code—14

**MATHEMATICS**

*Time : 3 Hours*

*Maximum Marks : 150*

**Note :** Attempt any *Five* questions. All questions carry equal marks. Q. No. 1 is compulsory. Answer *two* questions from Part I and *two* questions from Part II. The parts of the same question must be answered together and must not be interposed between answers to other questions.

1. Answer any *four* of the following : ( $4 \times 7\frac{1}{2} = 30$ )

- (a) Let  $\{e_1, e_2, e_3, e_4\}$  be a basis for a vector space  $V$  over  $\mathbf{R}$ . Prove that  $\{e_1 - e_2, e_2 - e_3, e_3 - e_4, e_4 - e_1\}$  is also a basis of  $V$ .

P.T.O.

(b) Let  $g : \mathbf{R} \rightarrow \mathbf{R}$  be defined by :

$$g(t) = 0 \text{ if } t \text{ is irrational or } 0$$

$$= \frac{1}{n} \text{ if } t = \frac{m}{n}$$

where  $m$  and  $n$  are integers,  $t$  is non-zero and highest common factor of  $m$  and  $n$  is 1.

Prove that  $g$  is continuous at all irrational  $t$  and discontinuous at all rational non-zero  $t$ .

(c) Find the equation of the plane passing through the line :

$$\frac{x-1}{4} = \frac{y-2}{6} = \frac{z-1}{3}$$

and the point (4, 3, 7).

(d) Let ABCD be a square. Suppose forces represented in magnitude and direction by AB, 2BC, 2CD, DA and DB are acting at a point O. Prove that they are at equilibrium.

- (e) A truck is moving along a level road at the rate of 40 km/hr. In what direction a bullet must be fired from it with a velocity of 200 m/sec so that its resultant motion is perpendicular to the truck ?
- (f) A random variable  $x$  follows Poisson distribution such that  $P(x = 1)$  is equal to  $P(x = 2)$ . Find  $P(x > 3)$ .

**Part I**

2. (i) Let  $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$  be the linear transformation defined by :

$$T(x, y) = (2x + 3y, y + 3x)$$

Find the matrix of  $T$  with respect to the basis  $\{(1, 1), (1, -1)\}$  (10)

- (ii) Find the matrix  $P$  such that  $P'AP$  is diagonal where  $P'$  denotes the transpose of  $P$  and  $A$  is the matrix : (12)

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 3 \\ -1 & 3 & 1 \end{bmatrix}$$

(iii) Let  $\lambda$  and  $\mu$  be distinct eigen values of a Hermitian matrix  $H$ . Suppose  $x$  and  $y$  are eigen vectors corresponding to  $\lambda$  and  $\mu$  respectively. Prove that  $x$  and  $y$  are mutually orthogonal. (3)

3. (i) Prove that if  $f : [0, 1] \rightarrow \mathbf{R}$  is continuous on  $[0, 1]$  except at finitely many points, then  $f$  is Riemann integrable. (10)

(ii) Find the volume of the torus generated by revolving the circle :

$$x^2 + y^2 = 4$$

about the line  $x = 3$ .

(iii) Determine the points of local maximum or minimum of the function :

$$x^3 + y^3 - 3xy^2$$

has a maximum or minimum. (5)

(iv) Find the radius of curvature of the curve :

$$x^{(2/3)} + y^{(2/3)} = a^{(2/3)}$$

at the point  $(a \cos^3\theta, a \sin^3\theta)$ . (5)

4. (i) Solve the differential equation : (8)  
 $y \sin 2x \, dx - (1 + y^2 + \cos x) \, dy = 0.$

(ii) Solve :

$$(D^2 + a^2) y = \sin ax. \quad (12)$$

(iii) Solve :

$$(2x^2y - 3y^4)dx + (3x^2 + 2xy^3)dy = 0. \quad (10)$$

### Part II

5. (i) Find curl grad F, where  $F = x^2y + 2xyz + z^2.$  (8)
- (ii) If  $r$  and  $a$  are two vectors, prove that  $\text{curl}(r \times a) = -2a.$  (7)
- (iii) State Gauss's divergence theorem and use it to evaluate

$$\iiint_S x^2 \, dx \, dz + y^2 \, dz \, dx + 2z(xy - x - y) \, dx \, dy$$

where  $S$  is the surface of the cube  
 $0 \leq x \leq 1, 0 \leq y \leq 1$  and  $0 \leq z \leq 1.$

(15)

6. (i) Prove that a continuous real valued function defined on  $[3, 8]$  is uniformly continuous on  $[3, 8].$  (10)