Code-14

MATHEMATICS
Time: 3 Hours.
Maximum Marks: 150

Note: Attempt Five questions in all. All questions carry equal marks. Question No. 1 is compulsory. Answer two questions from part I and two questions from part II. The parts of the same question must be answered together and must not be interposed between answers to other questions.

1. Answer any four of the following: (4x7.5=30)

   (a) Show that the set of all polynomials with real coefficients and degree not exceeding m is a vector space.

   (b) Find the dimensions of the vector space of part (a).

   (c) Let S be the set of solutions of $y''- 4y = 8$. Examine whether S is a vector space or not.

   (d) Describe a physical situation and write down its mathematical formulation.

   (e) Name few persons who have made substantial contribution in vector analysis. Is there any relationship between directional derivative and gradient? Prove that divCurl $\vec{F}$ =0, where $\vec{F}$ is a continuous vector field.

   (f) Prove that every differential function is continuous but the converse need not be true.

Part I

2(a) The Population of a city equals 60,000 at the beginning of the year 2007 and is growing continuously at a yearly rate of 5%.

   (i) Determine the population of the city at the beginning of the year 2017. (7.5)

   (ii) Calculate the time after which the size of the population will have doubled since 2007. (7.5)

(b) In the skull of an animal found in an archaeological dig, it was determined that about 20% of the original amount of carbon-14 was still present. The half-life of carbon-14 is 5,600 years. Find the approximate age of the animal that is, the period during which the animal lived. (15)

3(a) Find the work required to compress a spring from the natural length of 0.8 meters, if the force constant k = 16 kg/m. (10)

(b) A flat isosceles right triangular plate with base 6ft and height 3ft is submerged vertically base up, 2ft below the surface of a swimming pool. Find the force exerted by the water against one side of the plate. (10)

(c) Find the slope of the tangent line to the circle $r = 4\cos\theta$ at the point where $\theta = \pi/4$ (10)

4(a) Consider the differential equation $y - x\frac{dy}{dx} = 0$

(i) Show that the equation is not exact

ii) Find an integrating function that is a function of x alone (13)

(b) Suppose we have to design a 1 litre oil can shaped like a right circular cylinder. How should we choose radius r and height h in order to use the least amount of material? (8)

(c) A rectangle is to be inscribed into a semicircle of radius 2 cm. What is the largest area that the rectangle can have and what are its dimensions. (9)

Part II

(5) (a) Prove that every continuous function defined on [a, b] attains its minima and maxima. (10)

(b) Verify the statement of the Gauss divergence theorem with the data :

$\vec{F}(x, y, z) = x\vec{i} + y\vec{j} + z\vec{k}$, where S is piecewise smooth closed surface consisting of the surface S_1 of the cone $Z = \sqrt{x^2+y^2}$ for $x^2+y^2 \leq 1$, together with the flat cap S_2 consisting of the disc $x^2+y^2 \leq 1$ in the plane $z = 1$. (20)

6(a) The Philips company manufactures a 200 W light bulb. Laboratory tests showed that the life span of these light bulbs have a distribution described by the probability density function $f(x)=0.01e^{-0.001x}$. Determine the probability that a light bulb will have a life span of

(i) 500 hrs or less

(ii) More than 500 hrs (10)

(iii)More than 1000 hrs. but less than 1500 hrs.

(b) Find eigen values and corresponding eigen vectors of the following matrix (10)

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 0 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

(c) Discuss convergence and divergence of the series (10)

$\Sigma_{n=1}^{\infty}(-1)^n \frac{1}{n^p}$ where p is a real number. Show that the series

$1+\frac{1}{3\sqrt{2}} + \frac{1}{3\sqrt{3}} + \ldots\ldots\ldots\ldots + \frac{1}{3\sqrt{n}} + \ldots\ldots\ldots\ldots$ diverges.

7(a) Describe the Newton - Raphson method and apply it to find approximate value of $\sqrt{5}$. (10)

(b) Determine the velocity, speed, acceleration and tangential and normal components of the acceleration of the following position vector (10)

$\vec{F} = (e^t \sin t)\ \vec{i} + (-1)\vec{j} + (e^t \cos t)\vec{k}$.

(c) Find a least squares line or the regression line for the following data points (10)
(1,1), (2,1) and (3,3).